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MATHEMATICAL SIMULATION OF HEAT-AND-MASS TRANSFER IN GAS HYDRATE DEPOSITS IN A HIGH-FREQUENCY ELECTROMAGNETIC FIELD

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The laboratory and field experiments of [1, 2] showed the possibility of using a high-frequency electromagnetic (EM) field for deep and intense heating of a productive bed by heat sources distributed over the volume. The heat sources occur during interaction of a high-frequency EM radiation with the medium and are caused by conversion of part of energy of the propagating EM waves to heat. When a hydrate-containing rock is heated to the temperature of thermal decomposition of the gas hydrate, the latter can dissociate to the gas and water. Owing to the heat sources distributed over the volume, the phase transition can also occur in the absence of a temperature gradient. In this case, vast zones of phase transition can occur in which the gas hydrate decomposition temperature is attained and the EM field energy is expended on its dissociation.

In [3–5], the Stefan problem was used as a mathematical model for the mathematical description of processes that occur in a heated medium and are accompanied by phase transitions (melting and decomposition). However, the assumption that phase transitions proceed on a geometrical surface (a front of zero thickness) is applicable only in the case where the width of the phase-transition zone is much smaller than the length of the EM waves radiated into the bed [3]. In addition, the width of the phase-transition zone should be much smaller than the characteristic dimension of the problem, for example, the characteristic length of the zone in which the EM radiation is absorbed by the medium. These conditions are satisfied for small times of heating. It has been shown [5] that, as the productive bed is heated by the EM field, the width of the phase-transition zone increases rapidly and, at some value of the width, the use of the Stefan mathematical model gives a distorted picture of real processes.

The expression used in [3–5] for the density of heat sources distributed near the EM-wave radiator gives a value that is more than 2 times larger than the real value calculated from the exact solution expressed in terms of Hankel functions.

In this connection, it is necessary to use a phase-transition zone of finite width in the mathematical model and obtain new expressions for the distribution of heat sources.

1. System of Equations Describing the Thermodynamics of Gas Hydrate Decomposition under the Action of a High-Frequency EM Field. We study the following problem. A hydrate-saturated rock is under a bed pressure at a temperature lower than the decomposition temperature of the gas hydrate at the given pressure. The pore space is initially filled with the gas and the gas hydrate.

At the borehole bottom, a sufficiently powerful source of high-frequency EM waves is located opposite to the productive bed. As the EM waves propagate, their energy is converted to heat. Over a fairly large volume of the borehole bottom, the temperature increases, and, near the radiator, it reaches the gas hydrate decomposition temperature that corresponds to the bed pressure. A moving boundary (or an extended region) of the phase transition appears.

Maksimov and Tsypkin [6] noted that the gas hydrate decomposition regime in the bed depends on the permeability and, at low permeability of the productive bed, volume zones in which the hydrate and its decomposition products are present cannot exist simultaneously. Therefore, first of all, it is of interest to consider primarily hydrate-saturated rocks with high permeability.

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Because of the high permeability of the bed and the adopted simplifying assumptions that water filtration is absent and the gas viscosity is small, the pressure gradient is small. Therefore, the phase-transition temperature of the gas hydrate changes insignificantly. It is reasonable to assume that the phase-transition temperature practically does not change during the mining of the bed and extends over the entire volume zone of the phase transition, i.e., the energy of the high-frequency EM field in the transition region is expended only on the hydrate decomposition.

We assume that the frame of the porous bed and the gas hydrate are incompressible, water is an incompressible liquid, the gas satisfies the Clapeyron equation, and the capillary effects are small. The entire process of action of the high-frequency EM field s on the hydrate-containing bed can be divided into three stages in the mathematical formulation: in the first stage, the bed is heated from the initial state to the beginning of the phase transition at the borehole bottom and the occurrence of a moving decomposition boundary; in the second stage, the Stefan problem is solved, i.e., the assumption of the progress of the phase transition on a geometrical surface — the front of zero thickness; in the second stage, the non-Stefan problem is solved, i.e., the decomposition of the gas hydrate in the zone of finite thickness located between the region of completely decomposed gas hydrate and the region where the phase transition has not yet begun.

It is assumed that, in the first stage of heating, filtration of the gas and water is absent and processes that occur under the action of the EM field on the gas hydrate bed are described only by the heat-propagation equation

$$c\rho \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda \frac{\partial T}{\partial r}\right) + q, \quad c\rho = (1-m)\,c_0\rho_0 + m\,(\sigma_1\rho_1 c_{p1} + \nu c_3\rho_3),$$

$$\nu + \sigma_1 = 1, \quad \rho_1 = \rho_{10}\frac{P}{P_0}\frac{T_0}{T}\frac{1}{z}, \quad \lambda = (1-m)\lambda_0 + m\,(\sigma_1\lambda_1 + \nu\lambda_3)$$

subject to the boundary conditions

$$T(r,0) = T_p, \quad T(\infty,t) = T_p, \quad \frac{\partial T(r_0,t)}{\partial r} = 0, \quad \sigma_1(r,0) = \sigma_{10}, \quad \nu(r,0) = \nu_0,$$

where c and ρ are the specific heat and specific density, T is the temperature, t is time, r is the current coordinate, λ is the heat conductivity, q is the density of heat sources, m is the porosity, σ_1 is the gas saturation, c_{p1} is the specific heat at constant pressure, ν is the hydrate saturation, ρ_{10} is the gas density under normal conditions ($P_0 = 0.1$ MPa and $T_0 = 273$ K), P is the pressure, z is the coefficient of supercompressibility of the gas, T_p is the initial temperature of the bed and the temperature of the rock around the bed, r_0 is the radius of the borehole, σ_{10} is the initial gas saturation, and ν_0 is the initial hydrate saturation; the subscripts 0, 1, and 3 refer to the frame of the rock, the gas, and the gas hydrate, respectively.

At the beginning of the gas hydrate decomposition, a moving boundary of the phase transition occurs $R(t)$, which divides the productive bed into regions I $[r_0 < r < R(t)]$ and II $[R(t) < r < \infty]$. It is assumed that, in the first stage of heating, the pressure at the borehole bottom decreases, and removal of the gas–water mixture formed begins. As in [6], it is assumed that, in the given stage of heating $\nu_0 < 1$, the processes that occur in region I are described by the system

$$c_{\rm I}\rho_{\rm I}\frac{\partial T_{\rm I}}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_{\rm I}\frac{\partial T_{\rm I}}{\partial r}\right) - \rho_1 c_{p1} V_1 \frac{\partial T_{\rm I}}{\partial r} + q_{\rm I}; \tag{1.1}$$

$$m\frac{\partial(\rho_1\sigma_1)}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho_1 V_1 r) = 0; \tag{1.2}$$

$$\rho_1 = \rho_{10}\frac{P}{P_0}\frac{T_0}{T}\frac{1}{z}, \quad \rho_2 = \text{const}, \quad \sigma_1 + \sigma_2 = 1; \tag{1.3}$$

$$V_1 = -\frac{K_0 K_1(\sigma_1)}{\mu_1}\frac{\partial P}{\partial r}; \tag{1.4}$$

$$c_{\rm I}\rho_{\rm I} = (1-m)\,c_0\rho_0 + m\,(\sigma_1\rho_1 c_{p1} + \sigma_2\rho_2 c_2); \tag{1.5}$$

$$\lambda_{\rm I} = (1-m)\,\lambda_0 + m\,(\sigma_1\lambda_1 + \sigma_2\lambda_2); \tag{1.6}$$

and the processes in region II are described by the system

$$c_{\mathrm{II}}\rho_{\mathrm{II}}\frac{\partial T_{\mathrm{II}}}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_{\mathrm{II}}\frac{\partial T_{\mathrm{II}}}{\partial t}\right)-\rho_1 c_{p1}V_1\frac{\partial T_{\mathrm{II}}}{\partial r}+q_{\mathrm{II}}; \tag{1.7}$$

$$m\frac{\partial(\rho_1\sigma_1)}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho_1V_1r)=0; \tag{1.8}$$

$$\rho_1=\rho_{10}\frac{P}{P_0}\frac{T_0}{T}\frac{1}{z},\qquad \sigma_1+\nu=1; \tag{1.9}$$

$$V_1=-\frac{K_0}{\mu_1}\frac{\partial P}{\partial r}; \tag{1.10}$$

$$c_{\mathrm{II}}\rho_{\mathrm{II}}=(1-m)\,c_0\rho_0+m\,(\sigma_1\rho_1c_{p1}+\nu c_3\rho_3); \tag{1.11}$$

$$\lambda_{\mathrm{II}}=(1-m)\,\lambda_0+m(\sigma_1\lambda_1+\nu\lambda_3). \tag{1.12}$$

Here V is the filtration velocity, σ_2 is the water saturation, K_0 is the absolute permeability of the bed, μ_1 is the gas viscosity, and $K_1(\sigma_1)$ is the relative permeability of the bed; the subscript 2 refers to water.

The boundary conditions for the second stage of heating can be adopted in the form [7]

$$\begin{gathered}P(r,0)=P_p,\quad P(r_0,t)=P_g,\quad P(\infty,t)=P_p,\quad \frac{\partial T_{\mathrm{I}}(r_0,t)}{\partial r}=0,\\ T_{\mathrm{II}}(\infty,t)=T_p,\quad T_{\mathrm{I}}(R,t)=T_{\mathrm{II}}(R,t)=T_*,\quad T_*=a\log P_*+b,\\ -\lambda_{\mathrm{I}}\frac{\partial T_{\mathrm{I}}(R,t)}{\partial r}+\lambda_{\mathrm{II}}\frac{\partial T_{\mathrm{II}}(R,t)}{\partial r}=m\rho_3\nu L\frac{dR}{dt},\quad V_{1+}-V_{1-}=m\frac{dR}{dt}\left(\frac{\theta\rho_3}{\rho_{1*}}\nu+\sigma_{1+}-\sigma_{1-}\right),\end{gathered} \tag{1.13}$$

where P_p and P_g are the initial and bottom pressure of the bed, T_* is the phase-transition temperature of the gas hydrate, a and b are empirical constant, L is the heat release due to the phase-transition of the hydrate, and θ is the mass concentration of the gas in the hydrate; the subscripts plus and minus refer to the quantities on the right and left at the dissociation front, and the asterisk refers to the quantities at the front that do not undergo a discontinuity.

After the width of the phase-transition zone has reached a value that cannot be ignored compared with the EM wavelength and the characteristic dimension of the problem, the third stage of heating begins. In this case, the problem is solved for the following three regions: region I [$r_0 < r < R_1(t)$], in which the phase transition has already been completed, $T_{\mathrm{I}} > T_*$, region II [$R_1(t) < r < R_2(t)$], in which the phase transition occurs and the gas, water, and hydrate coexist simultaneously, $T_{\mathrm{II}} = T_*$, and region III [$R_1(t) < r < R_2(t)$], in which the phase transition has not yet begun, $T_{\mathrm{III}} < T_*$. For the third stage of heating in region II, it is assumed that the hydrate saturation ν changes continuously from the value ν_0 at the point $r = R_2(t)$ to zero at the point $r = R_1(t)$.

The processes that occur in region I are described by system (1.1)–(1.6), and, in region III, they are described by system (1.7)–(1.12) with replacement of the subscript II by the subscript III. In the phase-transition zone (in region II), the processes are described by the system

$$\begin{gathered}c_{\mathrm{II}}\rho_{\mathrm{II}}\frac{\partial T_{\mathrm{II}}}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_{\mathrm{II}}\frac{\partial T_{\mathrm{II}}}{\partial r}\right)-\rho_1c_{p1}V_1\frac{\partial T_{\mathrm{II}}}{\partial r}+m\rho_3L\frac{\partial\nu}{\partial t}+q_{\mathrm{II}},\\ m\frac{\partial(\rho_1\sigma_1)}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho_1V_1r)=-m\rho_3\theta\frac{\partial\nu}{\partial t},\\ \rho_1=\rho_{10}\frac{P}{P_0}\frac{T_0}{T}\frac{1}{z},\quad \rho_2=\mathrm{const},\quad \sigma_1+\sigma_2+\nu=1,\quad V_1=-\frac{K_0K_1(\sigma_1)}{\mu_1}\frac{\partial P}{\partial r},\\ c_{\mathrm{II}}\rho_{\mathrm{II}}=(1-m)c_0\rho_0+m\,(\sigma_1c_{p1}\rho_1+\sigma_2c_2\rho_2+\nu c_3\rho_3),\\ \lambda_{\mathrm{II}}=(1-m)\,\lambda_0+m\,(\sigma_1\lambda_1+\sigma_2\lambda_2+\nu\lambda_3),\quad T_*=a\log P_*+b.\end{gathered} \tag{1.14}$$

The boundary conditions for the third stage are given only by the following expressions on the stationary boundaries $r = r_0$ and $r = \infty$:

$$P(r_0,t) = P_g, \quad P(\infty,t) = P_p, \quad \frac{\partial T(r_0,t)}{\partial r} = 0, \quad T_{\rm III}(\infty,t) = T_p.$$

On the moving boundaries, there are no boundary conditions because of the continuity of the hydrate saturation ν.

The absence of boundary conditions for $r = R_1$ and $r = R_2$ can be explained as follows. By analogy with formula (1.13), for $r = R_1$, we write

$$-\lambda_{\rm I}\frac{\partial T_{\rm I}(R_1,t)}{\partial r} + \lambda_{\rm II}\frac{\partial T_{\rm II}(R_1,t)}{\partial r} = m\rho_3\nu L\frac{dR_1}{dt}.$$

Since the temperature gradient in region II tends to zero because of the progress of the phase transition, and the hydrate saturation on the boundary $r = R_1$ also tends to zero, the second term on the left in the given equation and the right side tend to zero. As a result, the remaining term of the equation also tends to zero:

$$-\lambda_{\rm I}\frac{\partial T_{\rm I}(R_1,t)}{\partial r} \to 0.$$

For $r = R_2$, the boundary condition is

$$-\lambda_{\rm II}\frac{\partial T_{\rm II}(R_2,t)}{\partial r} + \lambda_{\rm III}\frac{\partial T_{\rm III}(R_2,t)}{\partial r} = m\rho_3 L(\nu_+ - \nu_-)\frac{dR_2}{dt}.$$

For the reason indicated above, the first term tends to zero. The right side of the equation also tends to zero, because the difference $(\nu_+ - \nu_-)$ on the boundary between the regions tends to zero. As a result, we obtain

$$\lambda_{\rm III}\frac{\partial T_{\rm III}(R_2,t)}{\partial r} \to 0.$$

Expressions that describe the distribution of heat sources in all three stages for each region are written in Sec. 3.

2. Estimate of the Width of the Phase-Transition Zone. Using the energy equation (1.14), which describes processes in the phase-transition zone, we estimate its possible width. This, in particular, will allow us to determine up to which heating time of the bed it is possible to use the Stefan problem as the mathematical model.

Because of the continuous variation in the hydrate saturation ν in the phase-transition zone, this region is an electrodynamically inhomogeneous medium, whose electrophysical characteristics depend on the coordinates.

We assume that the total heat from the distributed heat sources goes only for the gas hydrate decomposition, and, hence, a temperature gradient is absent in this region. Because of this, the energy equation (1.14) takes the form

$$m\rho_3 L\frac{\partial\nu}{\partial t} + q_{\rm II} = 0. \tag{2.1}$$

The solution of Eq. (2.1) is given by

$$\nu = \nu_0 - \int\limits_0^t \frac{q_{\rm II}}{m\rho_3 L}\,dt. \tag{2.2}$$

As the heat sources we use the known expression obtained for the radial distribution of EM waves for the far radiation zone:

$$q = \frac{\alpha N_0}{\pi r h}\exp\left(-2\alpha(r - r_0)\right). \tag{2.3}$$

Here α and N_0 are the damping factor and power of the EM waves and h is the thickness of the productive layer.

TABLE 1

R_1, m	R_2, m	d_f, m	t, day	R_1, m	R_2, m	d_f, m	t, day
0.0625	0.913	0.8508	13.7	4	21.4	17.4	1100
1	9.12	8.12	231	5	24.10	19.10	1458
2	14.3	12.3	490	10	34.75	24.75	3897
3	18.2	15.2	779				

We find the time of complete decomposition of the gas hydrate at the observed point $r = R_1$, by substituting (2.3) into (2.2):

$$t_1 = \frac{\nu_0 \pi R_1 h m \rho_3 L}{\alpha_{II} N_0} \exp(2\alpha_{II}(R_1 - r_0)).$$

To determine the time of onset of the gas hydrate decomposition t_2 at the same point, we ignore the conductive and convective terms in the energy equation (1.14) and also the heat losses into the rocks around the bed. Equation (1.14) takes the form

$$c_{II}\rho_{II} \frac{\partial T_{II}}{\partial t} = q_{II}.$$

Integrating this equation and using formula (2.3), we obtain

$$T_* = T_p + \frac{\alpha_{II} N_0 t_2}{\pi R_1 h c_{II} \rho_{II}} \exp(-2\alpha_{II}(R_1 - r_0)),$$

and, hence,

$$t_2 = \frac{(T_* - T_p)\,\pi R_1 h c_{II} \rho_{II}}{\alpha_{II} N_0} \exp(2\alpha_{II}(R_1 - r_0)). \tag{2.4}$$

Thus, the duration of the gas hydrate decomposition is given by the approximate relation

$$t_1 + t_2 = \frac{\pi R_1 h}{\alpha_{II} N_0} \exp(2\alpha_{II}(R_1 - r_0))(\nu_0 m \rho_3 L + (T_* - T_p)\, c_{II} \rho_{II}), \tag{2.5}$$

from which it is evident that the duration of the process, and, hence, the width of the phase-transition zone (we denote it by d_f) depend on many factors: the electrical and thermal parameters of the medium, the difference between the phase-transition temperature and the initial temperature of the bed, the porosity and thickness of the bed, the frequency, radius, and power of the radiator of EM waves, and the distance from the observed point to the EM-wave radiator.

If the time $t = t_1 + t_2$ for the given point $r = R_1$ is determined from formula (2.5) and the point $r = R_2$ for the same time t is determined from formula (2.4), one obtains the transcendental equation

$$R_2 = \frac{\alpha_{II} N_0 t}{(T_* - T_p)\,\pi h c_{II} \rho_{II}} \exp(-2\alpha_{II}(R_2 - r_0)).$$

Specifying various values for R_1, one can calculate the width of the phase-transition zone d_f (see Table 1). The parameter values used in the calculations are given in Sec. 4.

As noted above, the Stefan problem can be used as the mathematical model provided that the following two conditions are satisfied:

(1) the width of the phase-transition zone should be much smaller than the characteristic length of the problem;

(2) the width of the phase-transition zone should be much smaller than the length of the EM waves λ_p radiated into the productive bed.

From the first condition, written as $d_f \ll 1/\alpha$, we obtain $d_f \ll 30$ m. The second condition requires $d_f \ll \lambda_p$. If we use the frequency $f = 13.56$ MHz of one commercial generator, then $d_f \ll 10$ m. The second condition imposes a more stringent requirement on the width of the phase-transition zone.

Analysis of Table 1 leads to the following conclusion. The Stefan problem can be used as the mathematical model only in the initial stage of heating, in which heating covers the borehole bottom of the bed. On further heating, the width of the phase-transition zone cannot be ignored.

The mathematical model described above allows one to study processes with a volume phase-transition zone that occur under the action of high-frequency EM fields on hydrate-saturated rocks with high permeability. This leads to the allowance for the gas fraction evolved in the decomposition of the gas hydrate in the transition region, whose width (see Table 1) can be significant. According to [7], with high permeability of the bed, the mass of the gas produced from the phase-transition zone can be several orders of magnitude larger than the mass of the gas produced from the region of complete dissociation of the hydrate.

3. Calculation of the Temperature-Field Distribution with Allowance for the Phase-Transition Zone. The temperature distribution in the productive bed is a basis for the calculation of the main technological characteristics of the thermal method of exploitation of gas-hydrate deposits (for example, the volume of the region heated to the phase-transition temperature, the amount of the gas evolved, the required power of the EM-wave radiator, etc.). In turn, the temperature distribution under volume heating of the medium is mainly determined by the distribution of heat sources, and the heat conductivity and convection only facilitate a more uniform temperature distribution in the medium.

For the mathematical simulation of the processes that occur in the decomposition of gas hydrates due to the action of a high-frequency EM field, the use of the phase-transition zone of nonzero width primarily requires obtaining new formulas for the power of the heat sources distributed in the productive bed. As noted above, the phase-transition zone is electrodynamically inhomogeneous medium, whose electrophysical characteristics depend on the coordinates. The mathematical problem of the field propagation in inhomogeneous media is very complicated and has no general solution. However, for the problem of a plane wave in a medium whose parameters depend on a single coordinate, there is a simple mathematical apparatus that provides, in some cases, an exact or approximate explicit solution [8, 9].

Electrodynamically, a gas hydrate bed is a nonmagnetic dielectric with losses. It is characterized by the complex dielectric permeability ε, which depends on the spatial coordinate. Let the EM field for the radial coordinate system depend on only one coordinate r. For a monochromatic EM wave, the wave equation is of the form [8]

$$\frac{d^2E}{dr^2} + \frac{1}{r}\frac{dE}{dr} + k^2\varepsilon(r)\,E = 0, \tag{3.1}$$

where E is the complex amplitude of the electric-field-strength component E_r (the subscript is omitted below), $k = \omega\sqrt{\varepsilon_0\mu_0}$ is the coefficient of EM-wave propagation in vacuum, ε_0 and μ_0 are the electrical and magnetic constants, and ω is the radial frequency.

The solutions of the wave equation (3.1) for region I ($r_0 \leqslant r < R_1$), in which the gas hydrate has completely decomposed and the medium is homogeneous, for region II ($R_1 < r < R_2$), in which the hydrate is still at the decomposition stage, and for region III ($R_2 < r < \infty$), in which the hydrate has not yet begun to decompose and the medium is homogeneous, can be obtained by the Wentzel–Kramers–Brillouin (WKB) method [8, 9]. For this, we represent the field strength E as the product of the slowly and rapidly changing factors (the amplitude and phase of the field) and substitute it into Eq. (3.1). Then, ignoring the second derivative of the slowly varying factor, we find the solution as the sum of the incident and reflected waves.

Using additional conditions, it is possible to obtain two simple equations that relate the unknown coefficients of the incident and reflected waves [8].

The electric-strength field is related to the EM-wave radiation conditions by the following formulas for the three regions, respectively:

$$E_{\rm I} = A\frac{1}{\sqrt{r}}\Big\{\exp(-jk\sqrt{\varepsilon_{\rm I}}r) + B\exp(jk\sqrt{\varepsilon_{\rm I}}r)\Big\}; \tag{3.2}$$

$$E_{\rm II} = \frac{A}{\sqrt{r}}\Big\{\frac{C_1(r)}{\sqrt[4]{\varepsilon_{\rm II}}}\exp\Big(-jk\int_{R_1}^{r}\sqrt{\varepsilon_{\rm II}}\,dr\Big) + \frac{C_2(r)}{\sqrt[4]{\varepsilon_{\rm II}}}\exp\Big(jk\int_{R_1}^{r}\sqrt{\varepsilon_{\rm II}}\,dr\Big)\Big\}; \tag{3.3}$$

$$E_{III} = AD\frac{1}{\sqrt{r}}\exp(-jk\sqrt{\varepsilon_{III}}r). \tag{3.4}$$

Here A is a general coefficient that depends on the conditions of EM-wave excitation in the medium, $j = \sqrt{-1}$ is the imaginary unity, B is the coefficient of reflected waves in region I, $C_1(r)$ and $C_2(r)$ are the coefficients of incident and reflected waves, respectively, in region II, and D is the coefficient of incident waves in region III.

Note that the exponents in formulas (3.2)–(3.4) are asymptotic approximations of Hankel functions in the far zone of the EM-wave radiation, in which the condition

$$k\sqrt{\varepsilon}r \gg 1 \tag{3.5}$$

is satisfied.

The coefficients $C_1(r)$, $C_2(r)$, and B can be refined iteratively [8]. As a first approximation for the coefficients $C_2(r)$ and B we have

$$C_2(r) = -\sqrt[4]{\varepsilon_I}\exp(jk\sqrt{\varepsilon_I}R_1)\int_r^{R_2}\frac{\dot{\varepsilon}_{II}}{4\varepsilon_{II}}\exp\Big(-2jk\int_{r_0}^{r}\sqrt{\varepsilon_{II}}\,dr\Big)\,dr,$$

$$B = -\int_{r_0}^{R}\frac{\dot{\varepsilon}_{II}}{4\varepsilon_{II}}\exp\Big(-2jk\int_{r_0}^{r}\sqrt{\varepsilon_{II}}\,dr\Big)\,dr,$$

where $\dot{\varepsilon}$ is the derivative with respect to the coordinate r, and it is also taken into account that $C_2(R_2) = 0$, i.e., there is no wave that propagates from the right.

The unknown coefficients $C_1(r)$ and D are found from the boundary conditions at the points $r = R_1$ and $r = R_2$:

$$C_1 = \sqrt[4]{\varepsilon_I}\exp(-jk\sqrt{\varepsilon_I}R_1), \qquad D = \frac{C_1}{\sqrt[4]{\varepsilon_{III}}}\exp\Big(jk\Big(\sqrt{\varepsilon_{III}}R_2 - \int_{R_1}^{R_2}\sqrt{\varepsilon_{II}}\,dr\Big)\Big).$$

The distribution of heat sources q in the medium can be determined from expression [4]

$$q = \frac{\omega\varepsilon_0\varepsilon'\tan\delta}{2}EE^* = \frac{\omega\varepsilon_0\varepsilon'\tan\delta}{2}|E|^2. \tag{3.6}$$

Here the superscript asterisk denotes the complex-conjugate quantity, ε' is the real part of the complex dielectric constant $\varepsilon = \varepsilon' - j\varepsilon''$, and $\tan\delta = \varepsilon''/\varepsilon'$ is the dielectric loss tangent.

The exponential factors in (3.2)–(3.4) are representable in terms of the damping factor α and the phase factor β [9]:

$$\begin{gathered}\exp(-jk\sqrt{\varepsilon}r) = \exp(-j(\beta - j\alpha)r),\\ \exp\Big(-jk\int_{R_1}^{r}\sqrt{\varepsilon_{II}}\,dr\Big) = \exp\Big(-j\int_{R_1}^{r}\beta_{II}\,dr - \int_{R_1}^{r}\alpha_{II}\,dr\Big).\end{gathered} \tag{3.7}$$

Taking into account formulas (3.6) and (3.7), we obtain the distribution of heat sources in the three regions in the form

$$q_I = \frac{\omega\varepsilon_0\varepsilon_I'\tan\delta_I}{2r}|A|^2\Big\{\exp(-2\alpha_I r) + |B|^2\exp(2\alpha_I r) + 2\mathrm{Re}(B^*\exp(-2j\beta_I r))\Big\}; \tag{3.8}$$

$$q_{II} = \frac{\omega\varepsilon_0\varepsilon_{II}'\tan\delta_{II}}{2r\sqrt{|\varepsilon_{II}|}}|A|^2\Big\{|C_1|^2\exp\Big(-2\int_{R_1}^{r}\alpha_{II}\,dr\Big)$$

$$+|C_2|^2\exp\Big(2\int_{R_1}^{r}\alpha_{II}\,dr\Big) + 2\mathrm{Re}\Big(C_1C_2^*\exp\Big(-2j\int_{R_1}^{r}\beta_{II}\,dr\Big)\Big)\Big\}; \tag{3.9}$$

$$q_{\rm III} = \frac{\omega\varepsilon_0\varepsilon'_{\rm III}\tan\delta_{\rm III}}{2r}|A|^2|D|^2\exp(-2\alpha_{\rm III}r), \tag{3.10}$$

where Re is the imaginary part of the complex quantity.

Using the expression for the Poynting vector and the formula for the magnetic-field strength H in the form [9]

$$H = Az^{-1}\frac{1}{\sqrt{r}}(\exp(-jk\sqrt{\varepsilon}r) - B\exp(jk\sqrt{\varepsilon}r)),$$

for the coefficient A we write

$$|A|^2 = \frac{N_0}{\pi h}\,\mathrm{Re}\left\{\frac{1}{z_{\rm I}^*}(\exp(-2\alpha_{\rm I}r_0) - |B|^2\exp(2\alpha_{\rm I}r_0) + 2j\,\mathrm{Im}(B\exp(2j\beta_{\rm I}r_0)))\right\}^{-1},$$

where $z_{\rm I}$ is the wave resistance of the medium in region I and Im is the imaginary part of the complex quantity.

In the third stage of heating in regions I–III, the heat sources q are given by relations (3.8)–(3.10); the dielectric characteristics of the saturated porous medium for the third stage are found from the following relations:

$$\varepsilon'_{\rm I} = (1-m)\,\varepsilon'_0 + m(\sigma_1\varepsilon'_1 + \sigma_2\varepsilon'_2), \quad \tan\delta_{\rm I} = (1-m)\tan\delta_0 + m(\sigma_1\tan\delta_1 + \sigma_2\tan\delta_2),$$

$$\varepsilon'_{\rm II} = (1-m)\varepsilon'_0 + m(\sigma_1\varepsilon'_1 + \sigma_2\varepsilon'_2 + \nu\varepsilon'_3), \quad \tan\delta_{\rm II} = (1-m)\tan\delta_0 + m(\sigma_1\tan\delta_1 + \sigma_2\tan\delta_2 + \nu\tan\delta_3),$$

$$\varepsilon'_{\rm III} = (1-m)\varepsilon'_0 + m(\sigma_1\varepsilon'_1 + \nu\varepsilon'_3), \quad \tan\delta_{\rm III} = (1-m)\tan\delta_0 + m(\sigma_1\tan\delta_1 + \nu\tan\delta_3).$$

In all the stages of heating, a uniform distribution of heat sources across the bed thickness and radial symmetry are assumed. In the first stage of heating, EM waves propagate in the medium without reflections, and the expression of heat sources is of the form [4]

$$q = \frac{\alpha\beta N_0}{\pi r_0 h}\,\frac{\left|H_0^{(2)}(k\sqrt{\varepsilon}r)\right|^2}{\mathrm{Re}\,(jk\sqrt{\varepsilon^*}H_0^{(2)}(k\sqrt{\varepsilon}r_0)H_1^{(2)*}(k\sqrt{\varepsilon^*}r_0))}, \tag{3.11}$$

where $H_i^{(1)}(\ldots)$ and $H_i^{(2)}(\ldots)$ ($i = 1$ and 2) are Hankel functions.

In the second stage, EM waves are partially reflected from the phase-transition boundary $R(t)$ (front of zero thickness), and the expressions of heat sources in regions I and II are of the form [4]

$$q_{\rm I} = \frac{\alpha_{\rm I}\beta_{\rm I}}{\omega\mu_0}|A|^2\left\{|H_0^{(2)}(k\sqrt{\varepsilon_{\rm I}}r)|^2 + |B|^2|H_0^{(1)}(k\sqrt{\varepsilon_{\rm I}}r)|^2\right\}$$

$$-\frac{|A|^2}{2}\left\{\mathrm{Re}\left(\frac{jB^*}{z_{\rm I}^*}\left(k\sqrt{\varepsilon_{\rm I}^*}H_0^{(2)}(k\sqrt{\varepsilon_{\rm I}}r)H_0^{(1)*}\left(k\sqrt{\varepsilon_{\rm I}^*}r\right) - k\sqrt{\varepsilon_{\rm I}}H_1^{(2)}(k\sqrt{\varepsilon_{\rm I}}r)H_1^{(1)*}\left(k\sqrt{\varepsilon_{\rm I}^*}r\right)\right)\right)\right.$$

$$\left.+\,\mathrm{Re}\left(\frac{jB}{z_{\rm I}^*}\left(k\sqrt{\varepsilon_{\rm I}^*}H_0^{(1)}(k\sqrt{\varepsilon_{\rm I}}r)H_0^{(2)*}\left(k\sqrt{\varepsilon_{\rm I}^*}r\right) - k\sqrt{\varepsilon_{\rm I}}H_1^{(1)}(k\sqrt{\varepsilon_{\rm I}}r)H_1^{(2)*}\left(k\sqrt{\varepsilon_{\rm I}^*}r\right)\right)\right)\right\},$$

$$q_{\rm II} = \frac{|A|^2|C|^2}{\omega\mu_0}\alpha_{\rm II}\beta_{\rm II}|H_0^{(2)}(k\sqrt{\varepsilon_{\rm II}}r)|^2,$$

$$B = \frac{z_{12}H_0^{(2)}(k\sqrt{\varepsilon_{\rm I}}R)H_1^{(2)}(k\sqrt{\varepsilon_{\rm II}}R) - H_0^{(2)}(k\sqrt{\varepsilon_{\rm II}}R)H_1^{(2)}(k\sqrt{\varepsilon_{\rm I}}R)}{H_0^{(2)}(k\sqrt{\varepsilon_{\rm II}}R)H_1^{(1)}(k\sqrt{\varepsilon_{\rm I}}R) - z_{12}H_0^{(1)}(k\sqrt{\varepsilon_{\rm I}}R)H_1^{(2)}(k\sqrt{\varepsilon_{\rm II}}R)}, \quad z_{12} = \frac{z_{\rm I}}{z_{\rm II}}, \tag{3.12}$$

$$C = \frac{H_0^{(2)}(k\sqrt{\varepsilon_{\rm I}}R) + BH_0^{(1)}(k\sqrt{\varepsilon_{\rm I}}R)}{H_0^{(2)}(k\sqrt{\varepsilon_{\rm II}}R)},$$

$$|A|^2 = \frac{N_0}{\pi r_0 h}\,\mathrm{Re}\left\{\frac{j}{z_{\rm I}^*}\left(H_0^{(2)}(k\sqrt{\varepsilon_{\rm I}}r_0)H_1^{(2)*}\left(k\sqrt{\varepsilon_{\rm I}^*}r_0\right) + B^*H_0^{(2)}(k\sqrt{\varepsilon_{\rm I}}r_0)H_1^{(1)*}\left(k\sqrt{\varepsilon_{\rm I}^*}r_0\right)\right.\right.$$

$$\left.\left.+\,BH_0^{(1)}(k\sqrt{\varepsilon_{\rm I}}r_0)H_1^{(2)*}\left(k\sqrt{\varepsilon_{\rm I}^*}r_0\right) + |B|^2H_0^{(1)}(k\sqrt{\varepsilon_{\rm I}}r_0)H_1^{(1)*}\left(k\sqrt{\varepsilon_{\rm I}^*}r_0\right)\right)\right\}^{-1}.$$

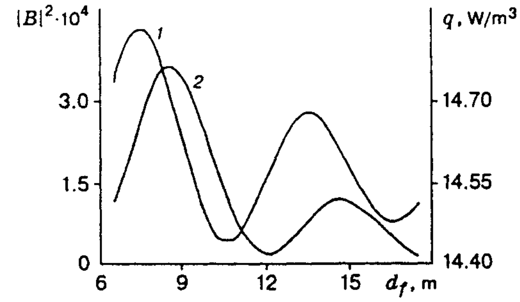

Fig. 1

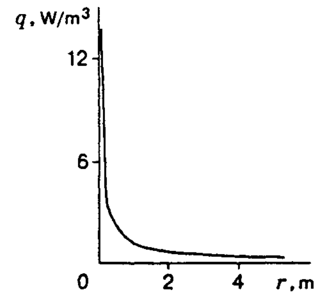

Fig. 2

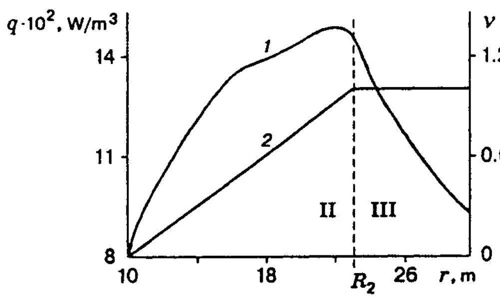

Fig. 3

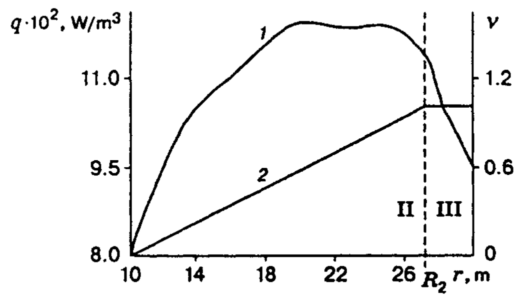

Fig. 4

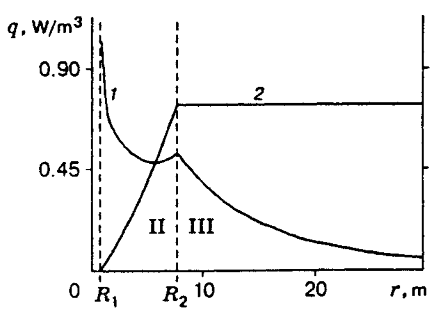

Fig. 5

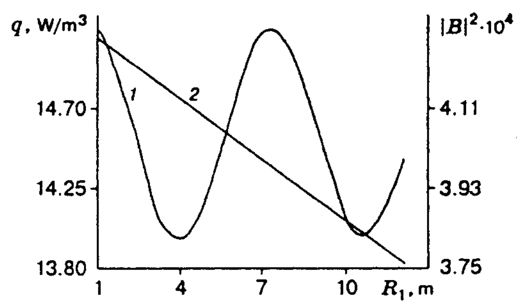

Fig. 6

When reflection of EM waves is absent, i.e., the reflection coefficient is $B = 0$, formula (3.12) becomes (3.11). If, instead of the Hankel function in formula (3.11), we write their asymptotic expressions for the far zone, in which condition (3.5) is satisfied, we obtain the known expression for distributed heat sources (2.3) [5]; (2.3) can be derived from formula (3.8), if we assume that $B = 0$ and $\tan\delta \ll 1$ and use the known expressions for the damping factors of EM waves α and the phase β [9]: $\alpha = k\sqrt{\varepsilon'}\tan\delta/2$ and $\beta = k\sqrt{\varepsilon'}$.

4. Results of the Numerical Studies. We give the results of calculations [using formulas (3.8)–(3.10)] of heat sources distributed in a productive bed including an inhomogeneous phase-transition zone of finite thickness for the following parameters: $m = 0.4$, $f = 13.56$ MHz, $N_0 = 10$ kW, $h = 10$ m, $r_0 = 0.0625$ m, $\varepsilon'_0 = 3.98$, $\tan\delta_0 = 0.02$, $\varepsilon'_1 = 1$, $\tan\delta_1 = 0.0001$, $\varepsilon'_3 = 4.17$, $\tan\delta_3 = 0.1$, $\varepsilon'_2 = 87$, $\tan\delta_2 = 0.002$, $\nu_0 = 1$, $\rho_3 = 800$ kg/m^3, $L = 5.1 \cdot 10^5$ J/kg, $T_p = 10$°C, $P_p = 20$ MPa, $P_q = 7$ MPa, $c_0\rho_0 = 1193$ kJ/(m$^3 \cdot$ K), $c_3\rho_3 = 2304$ kJ/(m$^3 \cdot$ K), and $\rho_2 = 1000$ kg/m^3. To simplify the calculations, we assumed $\sigma_2 = 0$.

The results obtained show the wave-like dependence of the reflection coefficient of EM waves, calculated from the power $|B|^2$ and density of heat sources q at the observed point, on the phase-transition zone width (region II). With increase in the width of region II, the vibration amplitude decreases, and, in the limit, $|B|^2$ tends to zero and q tends to its value for an infinite (without EM-wave reflection) medium.

The dependences $|B|^2(d_f)$ and $q(d_f)$ (curves 1 and 2) at the point of EM-wave radiation in region II are shown in Fig. 1 with a linear increase in the hydrate saturation ν from zero to unity.

A typical distribution of heat sources for the radial propagation of EM waves is shown in Fig. 2. In region I standing waves are not formed, and the density of heat sources is maximum at the borehole bottom. In region II, some formation of standing waves is observed.

The heat-source distributions in regions II and III for various values of the width of region II are shown in Figs. 3 and 4 (curves) with a linear increase in hydrate saturation (curves 2). In Fig. 4, the width of region II is larger than the EM-wavelength, and the formation of standing waves is evident. Obviously, for the decomposition of the hydrate mass that is not present in the porous medium, the formation of standing waves in region II will be more distinct.

In Figs. 3 and 4, region II is located far from the source of EM waves, and the radial character of wave propagation appears only slightly. Therefore, an increase in the density of heat sources with increase in hydrate saturation is observed. If region II is located near the source of EM waves, then, as shown in Fig. 5, despite the increase in hydrate saturation, the density of heat sources decreases exponentially in almost the entire region because of the radial propagation of EM waves.

Figure 6 shows the dependences of the density of heat sources at the borehole bottom (curve 1) and the reflection coefficient $|B|^2$ (curve 2) versus the position of the boundary R_1. In this case, R_2 varies in parallel with a change in R_1, but the difference between them remains constant. Evidently, the first dependence is wave-like (as the distance between region II and the EM-wave radiation source increases, the vibration amplitude decreases), and the second dependence decreases monotonically.

The aforesaid leads to the following conclusions. The investigation of the processes that occur under the action of an EM field on a productive bed leads to a qualitatively different class of physical problems, whose solution requires the introduction of a phase-transition zone of finite thickness into the mathematical model used. The thermodynamic state of the bed (temperature, pressure, and hydrate saturation) depends greatly on the width of the phase-transition zone, the location of its near and far boundaries relative to the radiation source, and the variation of these factors. This leads to nonlinearity of the energy equation that describes the temperature distribution in the productive bed.

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